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Propagation and reflexion of Alfvén–acoustic–gravity waves in an isothermal compressible fluid

By N. RUDRAIAH, M. VENKATACHALAPPA AND P. KANDASWAMY

Department of Mathematics, Central College, Bangalore University, India

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The propagation of internal Alfvén–acoustic–gravity waves in a compressible, stratified, inviscid, perfectly conducting, isothermal atmosphere in the presence of a horizontal magnetic field is investigated by considering both the horizontal and the vertical component of the group velocity. The vertical component of the group velocity is important because it determines the speed at which energy travels upwards and becomes available for heating the upper regions. The regions of propagation and no propagation of waves are delineated for different magnetic Mach numbers, in a refractive-index domain. The horizontal and vertical group velocities are compared with the corresponding phase velocity of the wave motion. It is found that the horizontal group velocity of the internal waves is always less than the horizontal phase velocity for small magnetic fields and vice versa for large magnetic fields, whereas the vertical group velocity is always opposite in direction to the vertical phase velocity for small magnetic fields and vice versa for large magnetic fields. We have also drawn the reflexion condition in a wave-number–frequency domain for different Mach numbers.

1. Introduction

Dessler & Parker (1959) have shown that geomagnetic storms result from fluctuations in the intensity of particles streaming from the sun, and that the disturbances are propagated through the ionosphere as hydromagnetic waves. Also, the extremely high temperature of the solar corona is generally believed to be due to the transfer of energy from the convection zone by waves. The major contribution to the heating of the solar corona comes from the internal Alfvén–acoustic–gravity waves. A full understanding of their role will depend in part on an understanding of the propagation conditions met at all levels in the region of propagation, in particular the part played by reflexion and ducting.

Recently Yu (1965) has discussed the propagation of internal Alfvén–acoustic–gravity waves in terms of three independent wave modes, namely the Alfvén mode, gravity mode and acoustic mode. However, in a magnetized plasma, it is in principle not correct to consider any one of these modes independently of the others. The modes all interact with each other and must be considered simultaneously. This has been discussed by McLellan & Winterberg (1968) in relation to heating of the solar corona but little attention has been paid to the physical mechanism of the propagation of such waves. Therefore the aim of the present analysis is to discuss in more detail the propagation, absorption and reflexion of these waves. For some observational purposes, the horizontal and vertical group velocities will be of greater consequence than the phase velocities. The vertical component of the group velocity is particularly important because it determines the speed at which energy travels upwards in the atmosphere and becomes available for heating the upper regions. Thus in this paper we have also made a detailed comparison of the group and phase velocities.

Recently Rudraiah, Venkatachalappa & Kandaswamy (1976) studied the propagation of internal Alfvén–acoustic–gravity waves in a non-uniform flow with particular attention to the transfer of energy and momentum from one region to another. However, for a complete understanding of these waves in the region of propagation and, in particular, the part played by reflexion and ducting, we must discuss the physical properties of these waves in more detail. This is done in this paper by considering, for mathematical simplicity, uniform basic flow. Specifically, we study the propagation of Alfvén–acoustic–gravity waves in a compressible, stratified, inviscid, perfectly conducting, isothermal atmosphere in the presence of a horizontal magnetic field. We consider a basic magnetic field whose magnitude varies with height in such a manner as to render the Alfvén velocity constant for the entire atmosphere. It may be remarked that in an actual atmosphere the density, pressure and magnetic field do change with height though not necessarily in the manner implied above. The assumptions of constant Alfvén velocity and constant temperature for the atmosphere are made for mathematical simplicity so as to evolve the simplest model of a hydromagnetic atmosphere, and it is hoped that the physics of the problem are not materially changed.

2. Mathematical formulation

We consider a system of Cartesian axes with the z axis in the vertical direction. We consider a fluid which is isothermal, compressible, inviscid and perfectly conducting with vertical density stratification. Under these assumptions the basic hydromagnetic equations governing the motion of the fluid are

$$\rho_1 D\mathbf{q}/Dt = -\nabla p_1 + \mu(\nabla \times \mathbf{H}) \times \mathbf{H} + \rho_1 \mathbf{g}, \tag{2.1}$$

$$D\rho_1/Dt + \rho_1(\nabla . \mathbf{q}) = 0, \tag{2.2}$$

$$Dp_1/Dt - c^2 D\rho_1/Dt = 0, \tag{2.3}$$

$$\partial \mathbf{H}/\partial t = \nabla \times (\mathbf{q} \times \mathbf{H}), \tag{2.4}$$

$$\nabla . \mathbf{H} = 0, \tag{2.5}$$

where

$$D/Dt = \partial/\partial t + (\mathbf{q} . \nabla), \tag{2.6}$$

$\mathbf{q}$ denotes the Eulerian velocity vector, ρ_1 the local fluid density, t time, μ the magnetic permeability, $\mathbf{H}$ the magnetic field, g the acceleration due to gravity and p_1 the hydrodynamic pressure.

2.1. *Equilibrium configuration*

The compressible ideal fluid is assumed to be at rest and the mass density $\rho_0(z)$ and the magnetic field $H_0(z)$, in the x direction, are assumed to be of the form

$$\rho_0(z) = \rho_c \exp(-\beta z), \quad H_0(z) = H_c \exp(-\tfrac{1}{2}\beta z), \tag{2.7), (2.8}$$

where β is the reciprocal of the scale height and is written as

$$\beta = g(c^2/\gamma + \tfrac{1}{2}A^2)^{-1} = g\gamma\{c^2(1+\tfrac{1}{2}\gamma M^2)\}^{-1},$$

where A denotes the Alfvén speed $(\mu H_0^2/\rho_0)^{\frac{1}{2}}$, $M = A/c$ is the magnetic Mach number and γ is the usual ratio of specific heats. For magnetostatic balance we have

$$dp_0/dz = -(g\rho_0 + \mu H_0\, dH_0/dz), \tag{2.9}$$

where p_0 denotes the steady-state hydrodynamic pressure.

2.2. *The perturbed state*

Upon the equilibrium configuration discussed above we superimpose a small disturbance of the form $(u, v, w), \rho_0+\rho, p_0+p, (H_0+h_x, h_y, h_z)$. Then the linearized forms of (2.1)–(2.6) admit plane-wave solutions in which any perturbation quantity, say f, may be written as

$$f = \mathrm{Re}[\hat{f}(z)\exp\{i(kx+ly-\sigma t)\}]. \tag{2.10}$$

Elimination of all variables but w leads to the wave equation

$$\frac{d^2\hat{w}}{dz^2} - \beta\frac{d\hat{w}}{dz} + \alpha^2\epsilon\hat{w} = 0, \tag{2.11}$$

where

$$\epsilon = \frac{(N^2+\Omega_A^2-\sigma^2)\,P + (\sigma^2-\Omega_A^2)\,(g^2k^2-N^2\sigma^2)}{(\sigma^2-\Omega_A^2)\,(Q-\sigma^4)}, \tag{2.12}$$

$$\alpha^2 = k^2+l^2,$$

$\Omega_c = \alpha c$ = sonic frequency, $\quad \Omega_A = kA$ = Alfvén frequency,

$$N^2 = g\beta,$$

$$P = (\sigma^2-\Omega_A^2)^2\,\Omega_c^2 + (l^2/k^2)\,\sigma^4\Omega_A^2,$$

$$Q = (\sigma^2-\Omega_A^2)\,(\sigma^2-\Omega_c^2) - (l^2/k^2)\,\sigma^2\Omega_A^2.$$

Equation (2.1) can be converted into the canonical form

$$d^2\psi/dz^2 + q^2\psi = 0 \tag{2.13}$$

by the change of variable

$$\hat{w} = \psi \exp\left(\int_0^z \tfrac{1}{2}\beta\, dz\right), \tag{2.14}$$

where

$$q^2 = \alpha^2\epsilon - \tfrac{1}{4}\beta^2. \tag{2.15}$$

The exponential factor in (2.14) gives rise to the well-known growth of amplitude with height. The q in (2.13) and (2.15) is the vertical wavenumber, which is a constant, m, say, in this isothermal atmosphere with constant Alfvén frequency Ω_A.

3. Propagation and reflexion of waves

In this section, using the packet velocity and phase velocity, we discuss in detail the condition for reflexion of waves and the region for evanescence. From (2.15), we find that real values of σ, k and l are accompanied by real values of q^2. An oversimplified but common approach is to view the waves as vertically propagating if $q^2 > 0$ and vertically evanescent if $q^2 < 0$ and to view any level at which $q^2 = 0$ as a reflexion level. In the following analysis, for simplicity we assume $l = 0$ (i.e. the axial orientation is chosen such that $\partial/\partial y$ vanishes).

3.1. *General characteristics of internal waves*

The dispersion relation (2.15) in the case $l = 0$ takes the form

$$q^2 = \frac{k^2c^2\omega_g^2 + \sigma^4}{(c^2+A^2)\,\sigma^2 - c^2\Omega_A^2} - k^2 - \frac{\beta^2}{4}, \tag{3.1}$$

where

$$\omega_g^2 = g\beta - \frac{g^2}{c^2} = \frac{g^2}{c^2}\left[\frac{\gamma}{1+\frac{1}{2}\gamma M^2} - 1\right].$$

Here ω_g is the Brunt–Väisälä frequency for the compressible conducting fluid in the presence of a magnetic field. Using the dimensionless quantities

$$Q = qc^2/g, \quad k_0 = kc^2/g, \quad \Omega_0 = l\sigma^2/(kg),$$

the dispersion equation (3.1) reduces to

$$Q^2 = k_0\left[\frac{\Omega_0^2 + \gamma/(1+\frac{1}{2}\gamma M^2) - 1}{(1+M^2)\,\Omega_0 - M^2k_0} - k_0 - \frac{\gamma^2}{4k_0\,(1+\frac{1}{2}\gamma M^2)^2}\right](1+\tfrac{1}{2}\gamma M^2)^2. \tag{3.2}$$

We confine our attention to real positive values of σ and k, i.e. the waves under consideration will be pure oscillatory in time and in the (horizontal) x direction. Then Q^2 is purely real and it follows that Q and m are either real (internal Alfvén waves) or purely imaginary (surface waves). In terms of the refractive-index components

$$n_x = kc/\sigma = (k_0/\Omega_0)^{\frac{1}{2}} \tag{3.3}$$

and

$$n_z = mc/\sigma = Q/(k_0\,\Omega_0)^{\frac{1}{2}}, \tag{3.4}$$

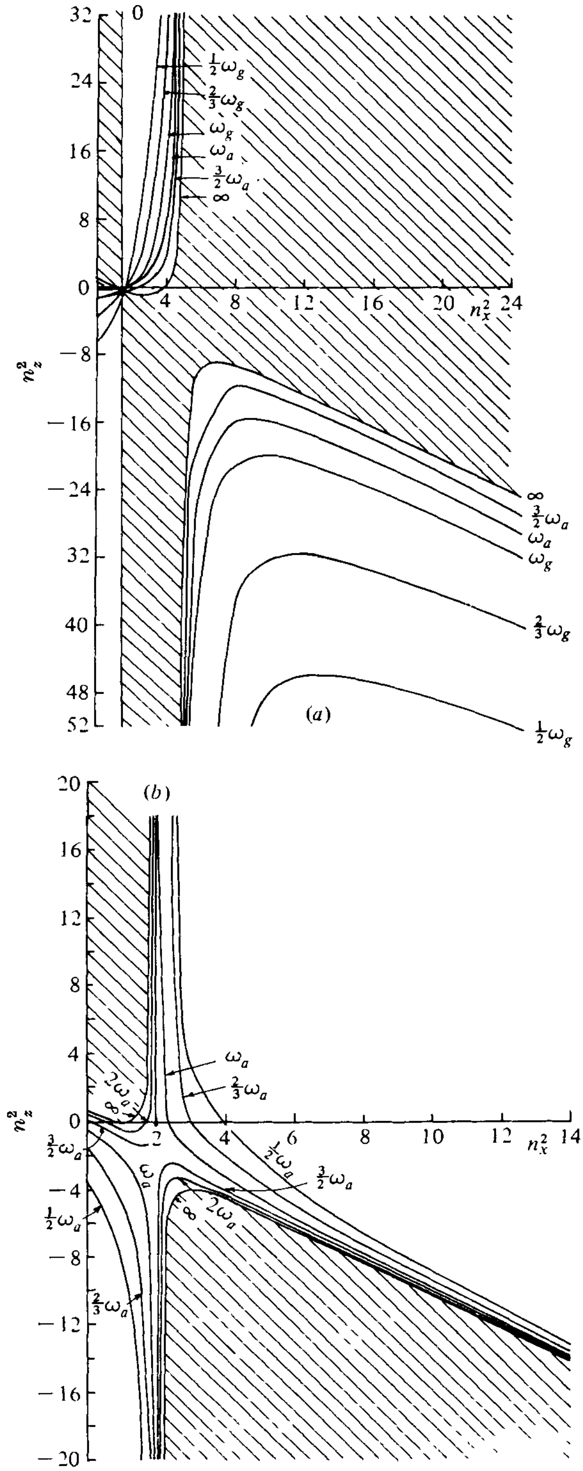
(3.2) may be written as

$$n_z^2 = \frac{1+(\omega_g^2/\sigma^2)\,n_x^2}{1+M^2-M^2n_x^2} - n_x^2 - \frac{\omega_a^2}{\sigma^2}, \tag{3.5}$$

where

$$\omega_a = g\gamma/\{2c(1+\tfrac{1}{2}\gamma M^2)\}. \tag{3.6}$$

Solutions of the modified dispersion equation (3.5) are plotted in figures 1 (*a*), (*b*) and (*c*) for the three cases $M = 0{\cdot}5$, 1 and 2 respectively. In these figures n_z^2 is



FIGURES 1 (*a*, *b*). For caption see next page.

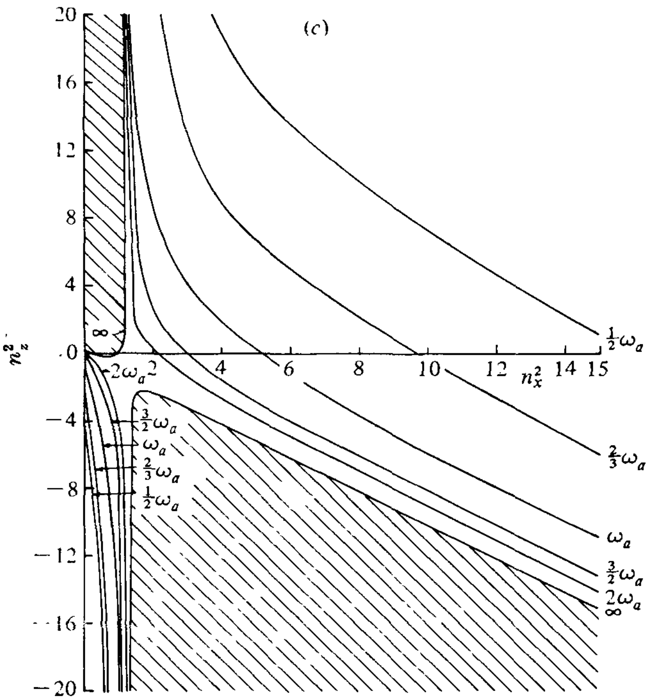


FIGURE 1. Alfvén–acoustic–gravity waves of constant period in the squared-refractive-index plane. Positive values of n_z^2 correspond to internal waves and negative values to surface waves. [hatched], region of no propagation. $\gamma = 1{\cdot}4$. (a) $M = 0{\cdot}5$. (b) $M = 1{\cdot}0$. (c) $M = 2{\cdot}0$.

plotted as a function of n_x^2 for waves of various constant periods, identified by the corresponding values of σ. In the hydrodynamic case (Pitteway & Hines 1965) these curves are straight lines, whereas in the present case they are hyperbolas, of which one branch passes through a common point (n_x^2, n_z^2) given by

$$n_x^2 = \frac{\omega_a^2(1+M^2)}{\omega_g^2+M^2\omega_a^2}, \quad n_z^2 = \frac{\omega_g^2+M^2\omega_a^2}{\omega_g^2(1+M^2)} - \frac{\omega_a^2(1+M^2)}{\omega_g^2+M^2\omega_a^2}.$$

Internal Alfvén–acoustic–gravity waves, with real n_z, correspond to the region above the n_x^2 axis ($n_z^2 > 0$), while surface waves correspond to negative values of n_z^2. There are no waves in the shaded region. From figures 1(a)–(c), we find that the region of no propagation (i.e. shaded region) changes as the magnetic field increases. In other words, from figures 1(a) and (b) we find that the region of propagation in the case $M = 0{\cdot}5$ becomes almost the region of non-propagation in the case $M = 1$. As the magnetic field increases, the Brunt–Väisälä frequency changes from positive to negative, i.e. there are no internal gravity waves for large magnetic fields. At this stage, it is of interest to compare the hydrodynamic and hydromagnetic waves. In hydrodynamics, there do not exist internal waves (Pitteway & Hines 1965) with real n_z (i.e. $n_z^2 > 0$) between the cut-off frequencies ω_g and ω_a, whereas, in hydromagnetics, for small magnetic fields there do exist (figure 1a) waves with real n_z between the frequencies ω_g and ω_a.

3.2. *Phase velocity and group speed*

The propagation of a wave form at the packet velocity or group speed will be of greater consequence than the phase motion. In particular, the vertical component of the group velocity is more important because it determines the speed at which energy travels upwards and becomes available for heating the upper regions. The packet or group velocity is generally given by

$$(\partial\sigma/\partial k, \quad \partial\sigma/\partial l, \quad \sigma/\partial m).$$

Here $\sigma = \sigma(k, l, m)$ is taken to be derived from the dispersion equation (5.6). In the present case $\partial\sigma/\partial l = 0$ since $l = 0$, and the horizontal and vertical components of the packet velocity are, respectively,

$$U_x = \frac{\partial\sigma}{\partial k} = \frac{V_x[\{(1+M^2-M^2n_x^2)^2-M^2\}\sigma^2-(1+M^2)\,\omega_g^2]}{[(1+M^2)/n_x^2-2M^2]\,\sigma^2-(1+M^2)\,\omega_g^2} \tag{3.7}$$

and

$$U_z = \frac{\partial\sigma}{\partial m} = \frac{V_z\sigma^2n_z^2\,(1+M^2-M^2n_x^2)^2}{n_x^2\,[(1+M^2)/n_x^2-2M^2]\,\sigma^2-(1+M^2)\,\omega_g^2}, \tag{3.8}$$

where $V_x = \sigma/k = c/n_x$ is the horizontal phase velocity and $V_z = \sigma/m = c/n_z$ is the vertical phase velocity. Equations (3.7) and (3.8) give the relations between the horizontal and vertical components respectively of the group and phase velocities. From these equations we find that the horizontal component U_x of the group velocity is zero when

$$\sigma^2 = \frac{(1+M^2)\,\omega_g^2}{(1+M^2-M^2n_x^2)^2-M^2}, \tag{3.9}$$

whereas the vertical component U_z of the group velocity is zero when

$$n_x^2 = (1+M^2)/M^2 \tag{3.10}$$

or

$$n_z^2 = 0. \tag{3.11}$$

Equation (3.9), using (3.5), yields

$$n_z^2 = \frac{1-M^2n_x^4}{1+M^2} - \frac{(1+M^2-M^2n_x^2)^2-M^2}{1+M^2} - \frac{\omega_a^2}{\omega_g^2}. \tag{3.12}$$

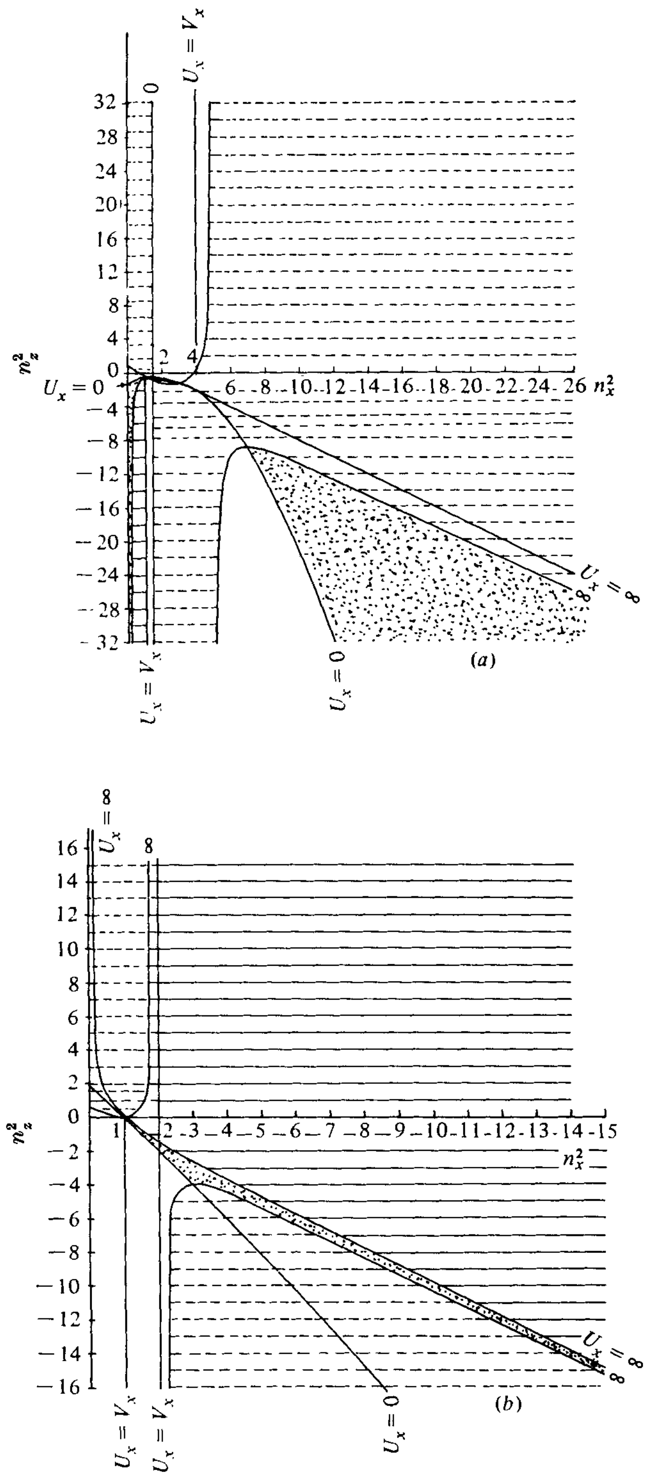
This reduces in the limit $M \to 0$ to the hydrodynamic condition of Pitteway & Hines (1965), namely

$$n_z^2 = -(\gamma-2)^2/4(\gamma-1). \tag{3.13}$$

The horizontal components of the group and phase velocities are equal (i.e. $U_x = V_x$) when

$$n_x^2 = 1,\ 1/M^2,\ (1+M^2)/M^2. \tag{3.14}$$

From (3.10) and (3.14) we observe that, along the line $n_x^2 = (1+M^2)/M^2$ in the n_x^2, n_z^2 plane, the vertical component of the group velocity will be zero and its horizontal component will be equal to the horizontal phase velocity. In other words, waves with horizontal refractive index $n_x^2 = (1+M^2)/M^2$ transfer energy in the horizontal direction only, with velocity equal to the horizontal phase velocity of the waves.



FIGURES 2 (a, b). For caption see next page.

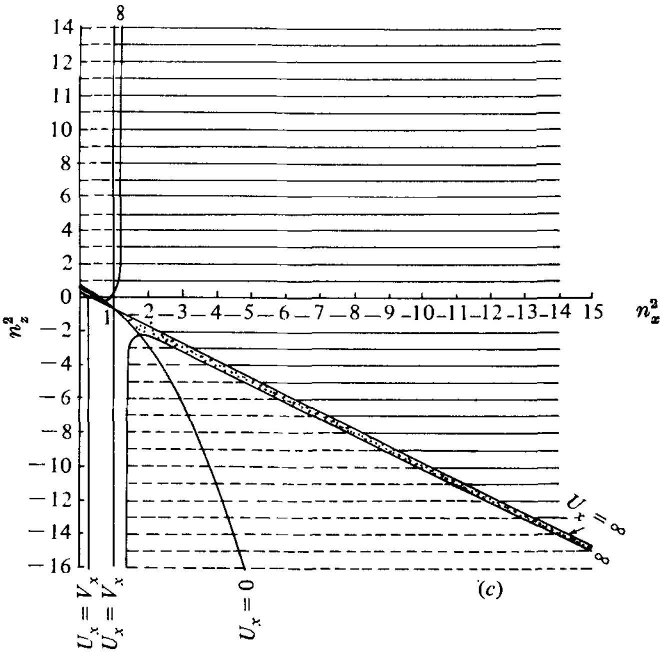


FIGURE 2. Relation between horizontal phase and group velocities in the squared-refractive-index plane. ▩, $V_x < 0$; ▤, $U_x > V_x$; □, $0 < U_x < V_x$; ▦, forbidden zone. $\gamma = 1{\cdot}4$. (*a*) $M = 0{\cdot}5$. (*b*) $M = 1{\cdot}0$. (*c*) $M = 2{\cdot}0$.

The vertical components of the group and phase velocities are equal (i.e. $U_z = V_z$) if

$$\sigma^2 = \frac{(1+M^2)\,\omega_g^2 n_x^2}{1+M^2-2M^2n_x^2-n_z^2(1+M^2-M^2n_x^2)^2}. \tag{3.15}$$

This condition, using (3.5), gives

$$n_z^2 = \frac{2/(1+M^2)-n_x^2\,[n_x^{-2}-2M^2/(1+M^2)]\,\omega_a^2/\omega_g^2}{2-\dfrac{M^2n_x^2}{1+M^2}-\dfrac{(1+M^2-M^2n_x^2)\,\omega_a^2}{n_x^2(1+M^2)\quad\omega_g^2}}. \tag{3.16}$$

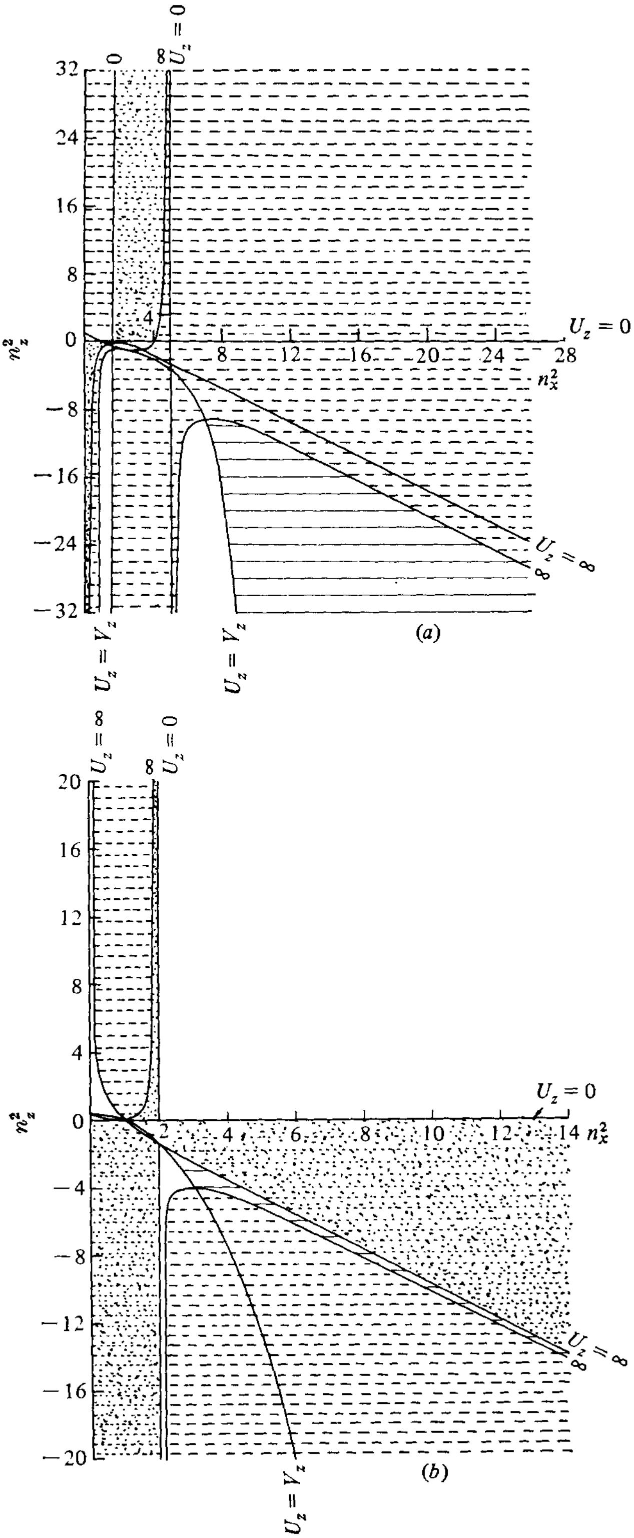
The horizontal and vertical components of the group velocity of the wave become infinite when

$$\sigma^2 = \frac{(1+M^2)\,\omega_g^2 n_x^2}{1+M^2-2M^2n_x^2}, \tag{3.17}$$

i.e., if the horizontal component of the group velocity of a wave becomes infinite, the vertical component will also be infinite. Condition (3.17), using (3.5), yields

$$n_z^2 = \frac{2}{1+M^2}\left(1+M^2\frac{\omega_a^2}{\omega_g^2}\right)-n_x^2-\frac{1}{n_x^2}\frac{\omega_a^2}{\omega_g^2}. \tag{3.18}$$

Using conditions (3.10)–(3.12) and (3.14)–(3.17), in figures 2 and 3 we have divided the n_x^2, n_z^2 plane into four regions of propagation or no propagation for the three cases $M = 0{\cdot}5$, 1 and 2. Figure 2 corresponds to the horizontal component of the



FIGURES 3 (*a*, *b*). For caption see next page.

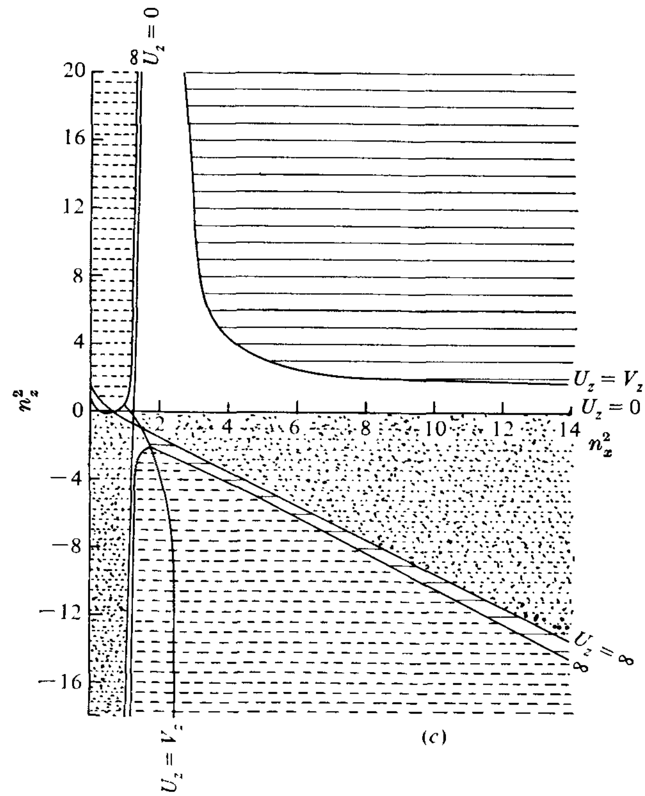


FIGURE 3. Relation between vertical phase and group velocities in the squared-refractive-index plane. ▨, $U_z < 0$; ▤, $U_z > V_z$; □, $0 < U_z < V_z$; ▤, forbidden zone. $\gamma = 1{\cdot}4$. (*a*) $M = 0{\cdot}5$. (*b*) $M = 1{\cdot}0$. (*c*) $M = 2{\cdot}0$.

group and phase velocities whereas figure 3 corresponds to their vertical components. The four regions are (i) $0 < U_x, U_z < V_x, V_z$, (ii) $U_x, U_z < 0$, so that the phase and group velocities are opposite in direction, (iii) $U_x, U_z > V_x, V_z$, i.e. the group velocity is greater than the phase velocity, and (iv) the region in which there are no Alfvén–acoustic–gravity waves, corresponding to the shaded regions in figure 1. From figure 2 it is evident that the horizontal components of the group and phase velocities are opposite in direction (i.e. $U_x < 0$) only for surface waves ($n_z^2 < 0$). This result is also true for hydrodynamic surface waves. For small magnetic fields we find (figure 2(*a*), $M = 0{\cdot}5$) that the region $U_x > V_x$ is confined to surface waves, but as the magnetic field increases (figures 2*b*, *c*) this region extends into both $n_z^2 > 0$ and $n_z^2 < 0$. However, the region $0 < U_x < V_x$ extends into both half-planes for small magnetic fields but as the magnetic field increases is confined to the lower half-plane $n_z^2 < 0$. From figures 1 (*a*)–(*c*) we observe that, as the magnetic field increases, the region of propagation for $M = 0{\cdot}5$ almost becomes the region of no propagation when $M = 1{\cdot}0$ or $2{\cdot}0$. The vertical components of the group and phase velocities of the internal waves ($n_z^2 > 0$) are in opposite directions ($U_z < 0$) for small magnetic fields (figure 3*a*) but are in the same direction for large magnetic fields (figures 3*b*, *c*). It is of interest to compare the horizontal and vertical velocities for small magnetic fields. Figures 2(*a*) and 3(*a*) show that the horizontal group and phase velocities of internal waves are in the same direction, whereas the vertical group and phase velocities are in opposite directions. For small magnetic fields the regions $U_z > V_z$ and $0 < U_z < V_z$ are confined to surface waves. As the magnetic field increases, these regions are mostly confined to internal waves. Also, for small magnetic fields, the group velocity is always positive for surface waves. For large magnetic fields the internal waves have positive vertical group velocity, whereas the surface waves have negative group velocity.

3.3. *Reflexion condition*

If the shear parameter k_0 varies only slowly with height, it is possible for an Alfvén–gravity wave to propagate upwards without reflexion until the vertical wavelength $2\pi/m$ becomes very large, reflexion occurring when $Q = 0$. Then the dispersion relation (3.2) becomes

$$\frac{\Omega_0^2+\gamma/(1+\frac{1}{2}\gamma M^2)}{(1+M^2)\,\Omega_0-M^2k_0} = k_0+\frac{\gamma^2}{4k_0\,(1+\frac{1}{2}\gamma M^2)^2}. \tag{3.19}$$

Equation (3.19) is the reflexion condition and its solutions are plotted in figures 4(*a*)–(*c*) for the three cases $M = 0{\cdot}5$, 1 and 2. The shaded areas in these figures mark the values of k_0 and Ω_0 that yield imaginary Q in (3.2) and thus correspond to surface waves. The corresponding region in the non-magnetic case lies between the dashed curves in figures 4(*a*)–(*c*). The two unshaded areas yield real values of Q, corresponding to internal Alfvén–acoustic–gravity waves. The solutions of (3.19) form the boundaries of the unshaded regions. Curves of constant frequency take the form of rectangular hyperbolas on these diagrams. The hyperbolas corresponding to $\sigma = \omega_a$ and $\sigma = \omega_g$ are given by

$$k_0\,\Omega_0 = \frac{\gamma^2}{4(1+\frac{1}{2}\gamma M^2)^2}, \quad k_0\,\Omega_0 = \frac{\gamma}{1+\frac{1}{2}\gamma M^2}-1,$$

where $\sigma = \Omega_A$ is a straight line given by $\Omega_0 = M^2k_0$. It is noted that in hydrodynamics the curves corresponding to $\sigma = \omega_a$ and $\sigma = \omega_g$ lie entirely in the shaded region, i.e. there exist only surface waves between these frequencies. However, in the present case, as is expected from § 3.1, we note that these curves do not lie entirely in the shaded region because of the existence of Alfvén wave sequences between these frequencies. For large magnetic fields there will be no internal waves, only Alfvén–acoustic waves.

In this paper we have assumed the atmosphere to be isothermal with constant Alfvén velocity. But in general an actual atmosphere will be non-isothermal and the Alfvén velocity will vary with height. In that case the parameters k_0 and Ω_0 will vary in a specified manner with height. Because of these variations, as a wave propagates in the vertical direction the parameters k_0 and Ω_0 will trace a locus in figures 4(*a*)–(*c*). At the height at which this locus intersects the boundaries of the shaded regions, the wave will be reflected.

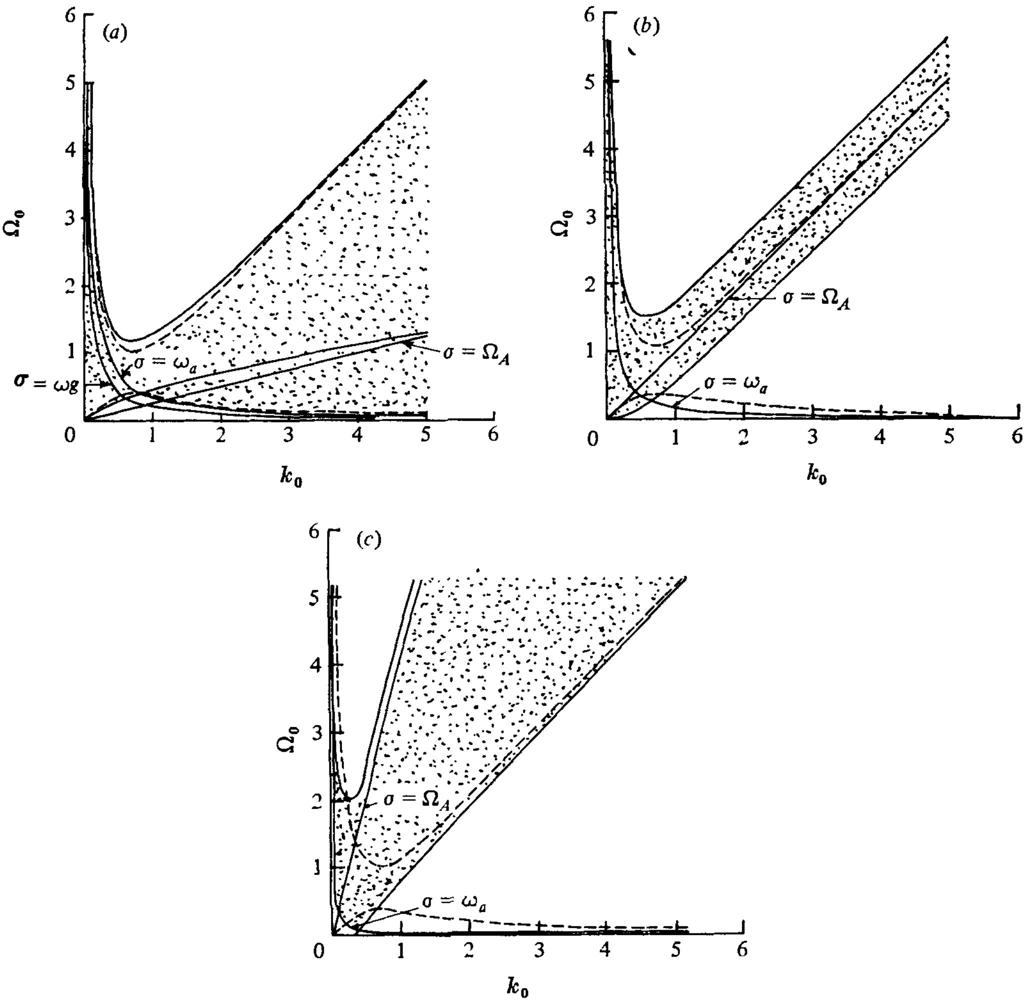


FIGURE 4. Reflexion conditions for Alfvén–acoustic–gravity waves. - - -, $M = 0$. (*a*) ——, $M = 0{\cdot}5$; (*b*) ——, $M = 1{\cdot}0$; (*c*) ——, $M = 2{\cdot}0$.

4. Conclusions

The characteristics of internal Alfvén–acoustic–gravity waves have been studied for a stationary conducting fluid. It has been found that (figure 1), as the magnetic field increases, the region of propagation for $M = 0{\cdot}5$ almost becomes the region of no propagation when $M = 1{\cdot}0$. Also, in the non-magnetic case there are no internal waves between the cut-off frequencies ω_g and ω_a, only surface waves, whereas in the magnetic case it is observed that there exist both internal waves ($n_z^2 > 0$) and surface waves ($n_z^2 < 0$) between these frequencies. As the magnetic field increases there will not be any internal gravity waves since ω_g becomes imaginary. The relation between the horizontal and vertical group and phase velocities of the wave motion was studied in §3.2. We have found that for small magnetic fields ($M = 0{\cdot}5$) the region $0 < U_x < V_x$ includes both internal and surface waves and the regions $U_x > V_x$ and $U_x < 0$ are confined to surface waves; whereas the regions $0 < U_z < V_z$ and $U_z > V_z$ are confined to surface waves and the region $U_z < 0$ to internal waves. In the cases $M = 1$ and 2 (figures 2*b*, *c*, 3*b*, *c*), the regions $0 < U_x < V_x$ and $U_x < 0$ correspond to surface waves and the region $U_x > V_x$ is mainly confined to internal waves, whereas the regions $U_z > V_z$ and $0 < U_z < V_z$ are mainly confined to internal waves ($n_z^2 > 0$) and the region $U_z < 0$ corresponds to surface waves. In other words, internal waves have positive vertical group velocity. Also, from figures 2(*a*)–(*c*) we observe that the horizontal group velocity of internal waves is always less than their phase velocity for small magnetic fields but is greater than their phase velocity for large magnetic fields. In §3.3 we have discussed the reflexion of Alfvén–acoustic–gravity waves. It has been observed, from figures 4(*a*)–(*c*), that for small magnetic fields the region of reflexion is almost the same as in the hydrodynamic case of Pitteway & Hines (1965). As the magnetic field increases, for a given wavenumber, the higher frequency waves propagate without reflexion.

The results of this analysis are of interest in connexion with the geophysical problem of the propagation of energy from the lower atmosphere to the upper atmosphere. More specifically, the analysis of the vertical component of the group velocity is useful in determining the speed at which energy travels upwards in the atmosphere and becomes available for heating the upper regions.

Finally, we conclude that the results of this paper are also applicable to an atmosphere moving with a uniform speed, because the case of a uniform flow is not different from that of gas at rest. It is only a question of making calculations relative to the moving gas, which reduces the problem to the case of gas at rest.

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